Electromagnetics

April 9, 2006

1. A *TEM* wave propagates within a coaxial structure shown in Fig. 1 with the fields given by

$$\mathbf{E} = \hat{\rho}E_\rho = \hat{\rho}\frac{E_0}{\rho}e^{-j\beta z}$$
$$\mathbf{H} = \hat{\phi}H_\phi = \hat{\phi}\frac{H_0}{\rho}e^{-j\beta z}.$$

Perfect conductors and lossless medium in between are assumed. Derive in details the time-harmonic transmission-line equations for V and I and the expressions for the associated line capacitance (C) and line inductance (L) of this coaxial structure through the two *Curl* equations of the time-harmonic Maxwell's equations. The *Curl* operation in general orthogonal coordinates is given below

$$\nabla \times \mathbf{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \hat{u}_1h_1 & \hat{u}_2h_2 & \hat{u}_3h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix},$$

where h_1, h_2, and h_3 are the metric coefficients of the coordinates (u_1, u_2, u_3). (20%)

2. An element shown in Fig. 2 is defined by the following surfaces:

- $r = r_1$ and $r = r_2$,
- $\theta = \frac{\pi}{2}$ and $\theta = \beta$,
- $\phi = 0$ and $\phi = \alpha$.

Compute the following if the material of the element is characterized by a conductivity of σ:

(a) the resistance of this element if the surface at $r = r_1$ has $V = 0$ and the surface at $r = r_2$ has $V = V_0$. Then, determine the inductance of this element if the material is characterized by a permeability of μ. Neglect fringing. (10%)

(b) the resistance of this element if the surface at $\theta = \dfrac{\pi}{2}$ has $V = 0$ and the surface at $\theta = \beta$ has $V = V_0$. Neglect fringing. (5%)

(c) the resistance of this element if the surface at $\phi = 0$ has $V = 0$ and the surface at $\phi = \alpha$ has $V = V_0$. Neglect fringing. (5%)

3. For two quarter circular line charges of density $2\rho_\ell$ and $-\rho_\ell$, respectively, located on the $x-y$ plane, as shown in Fig. 3, determine the following quantities at any point $(0, 0, z)$ on the $z-$axis,

   (a) the electric potential V, (5%)

   (b) the electric field intensity $\mathbf{E}$. (10%)

4. The electromagnetic fields in a rectangular waveguide shown in Fig. 4 are given by

$$\mathbf{E} = C\frac{\omega\mu_o a}{\pi}\sin\left(\frac{\pi x}{a}\right)\sin(\omega t - \beta z)\hat{y}$$

$$\mathbf{H} = -C\frac{\beta a}{\pi}\sin\left(\frac{\pi x}{a}\right)\sin(\omega t - \beta z)\hat{x} + C\cos\left(\frac{\pi x}{a}\right)\cos(\omega t - \beta z)\hat{z}$$

where C is a constant and $\omega = 2\pi f$, with f the frequency of excitation. The walls of the waveguide are assumed to be perfect conductors. Determine the surface charge densities and surface current densities on those walls. (15%)

5. Determine the following fields at a <u>remote</u> location $P(r, \theta, \phi)$:

   (a) the electric field intensity $\mathbf{E}$ caused by an electric dipole defined by its dipole moment $\mathbf{p} = qd\hat{z}$, as shown in Fig. 5(a), (5%)

   (b) the magnetic field intensity $\mathbf{B}$ caused by a magnetic dipole defined by its dipole moment $\mathbf{m} = I\mathbf{S} = I\pi a^2\hat{z}$, as shown in Fig. 5(b). (10%)

6. Consider an electromagnetic wave at oblique incidence on a dielectric interface, as shown in Fig. 6. Given the following conditions:

- $\epsilon_1 = 4\epsilon_0$ and $\epsilon_2 = \epsilon_0$,
- $\mu_1 = \mu_2 = \mu_0$,

determine the following:

(a) the *critical angles*, θ_c, for both polarizations, (5%)

(b) the angles of refraction of the refracted waves for both polarizations under the condition of $\theta_i > \theta_c$, (5%)

(c) and the *Brewster angles*, θ_B, for both polarizations. (5%)

You may express your answers in terms of the inverse trigonometric functions.

Laplace's equation in cylindrical coordinates:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

and in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0.$$

Some useful integrals:

$$\int \sin x dx = -\cos x + C, \quad \int \cos x dx = \sin x + C,$$

$$\int \tan x dx = -\ln(\cos x) + C, \quad \int \cot x dx = \ln(\sin x) + C,$$

$$\int \sec x dx = \ln\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] + C, \quad \int \csc x dx = \ln\left(\tan\frac{x}{2}\right) + C,$$

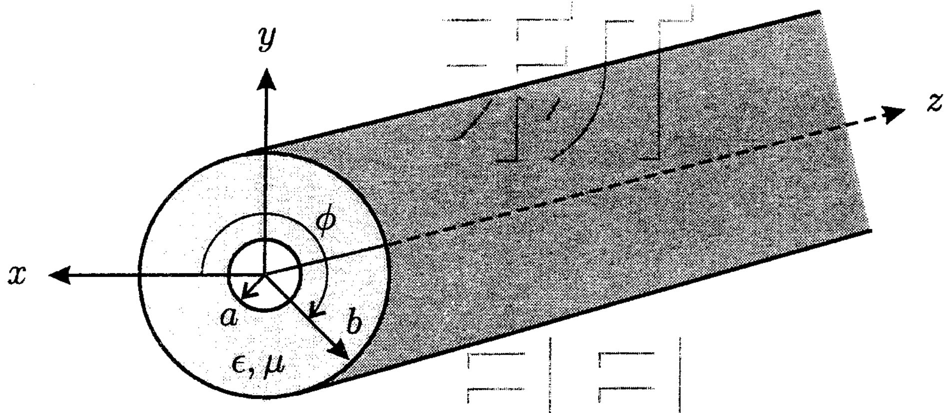


Figure 1: Figure for Problem 1.

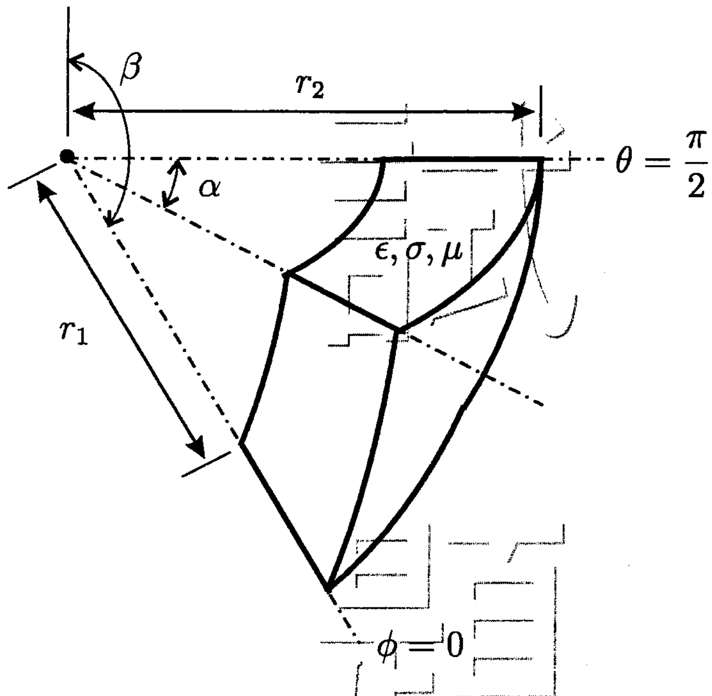


Figure 2: Figure for Problem 2.

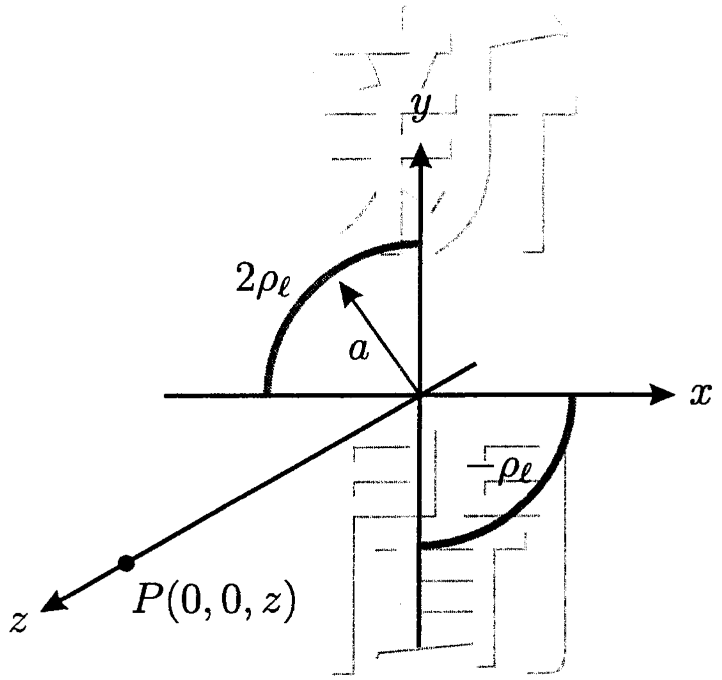


Figure 3: Figure for Problem 3.

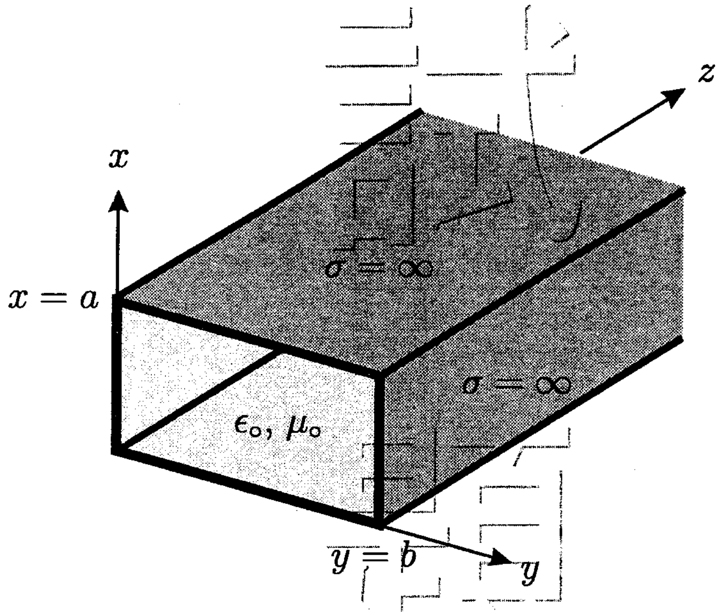
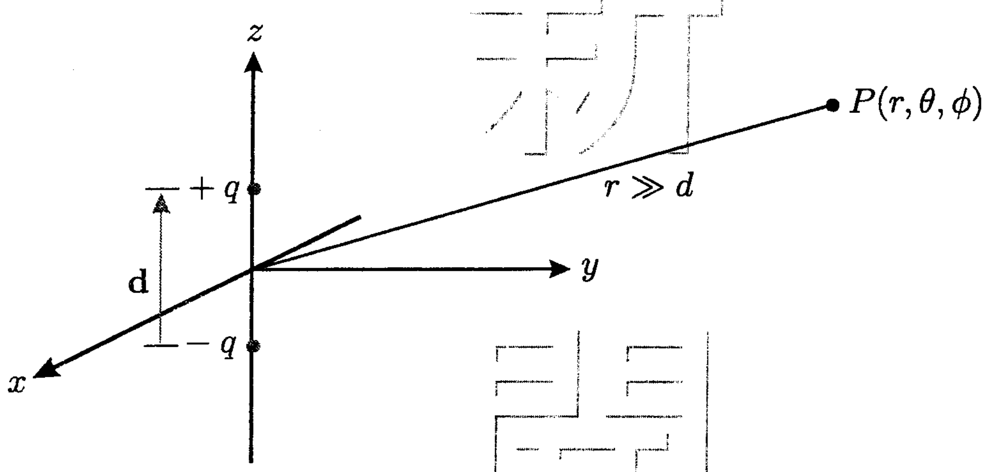
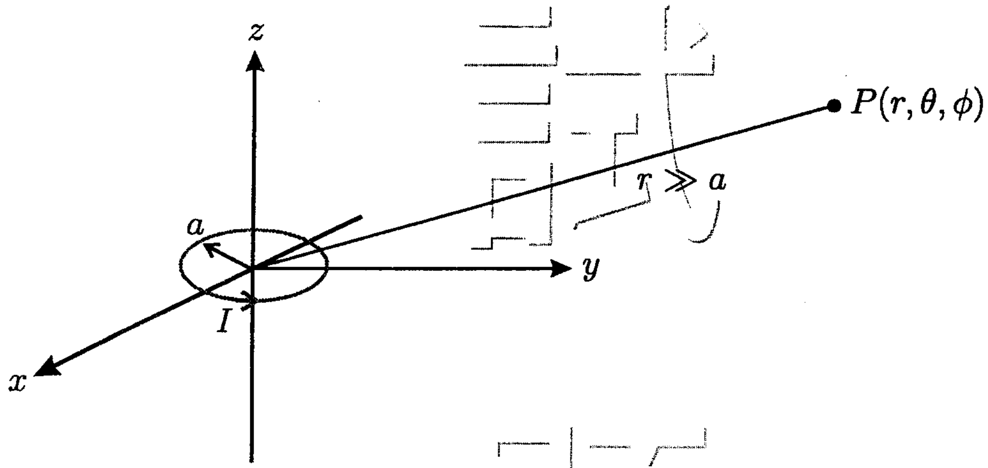


Figure 4: Figure for Problem 4.



5(a)



5(b)

Figure 5: Figures for Problem 5.

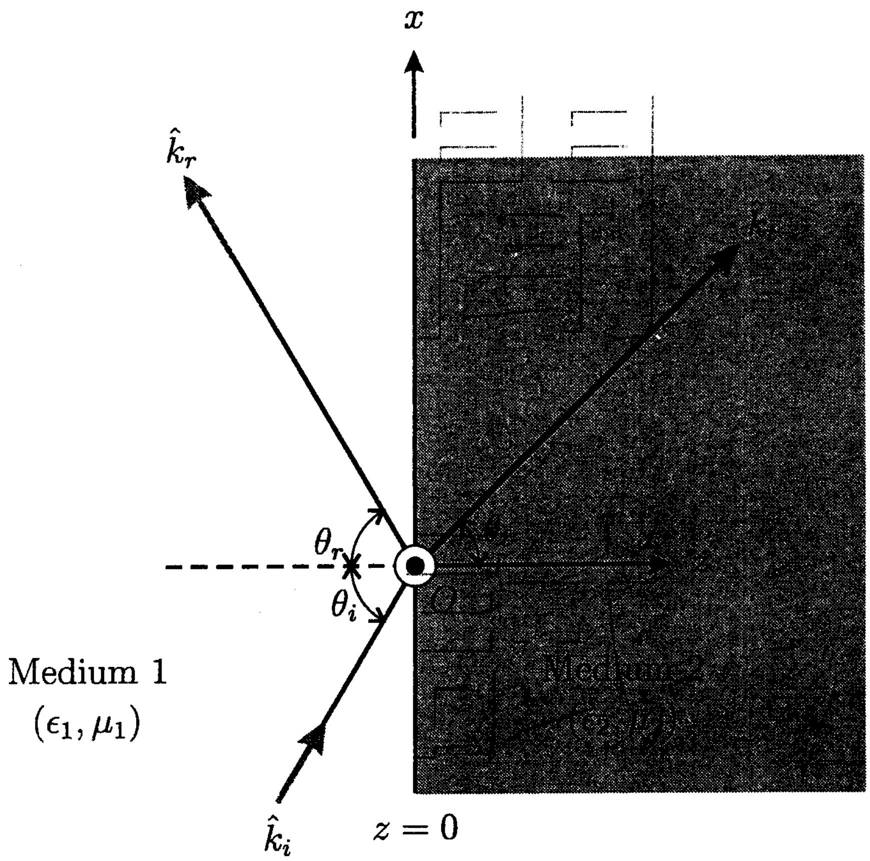


Figure 6: Figure for Problem 6.