1. (30 pts.) A linear time-invariant single-input-single-output physical system is described by the following ordinary differential equation with the output $y(t)$, the input $u(t)$ and some constant coefficients a_i, $i = 0, 1, 2, 3$, and b_j, $j = 0, 1, 2$:

$$\frac{d^4y(t)}{dt^4} + a_0\frac{d^3y(t)}{dt^3} + a_1\frac{d^2y(t)}{dt^2} + a_2\frac{dy(t)}{dt} + a_3y(t) = b_0\frac{d^3u(t)}{dt^3} + b_1\frac{d^2u(t)}{dt^2} + b_2\frac{du(t)}{dt} + u(t).$$

(a) This system can be represented with either ***transfer function*** or ***state variable*** models. Determine the input-output relationship of the system with these two types of models, and then compare the difference between the two representations. (20 pts.)

(b) Determine the coefficients a_i and b_j to guarantee the stability of this system. (10 pts.)

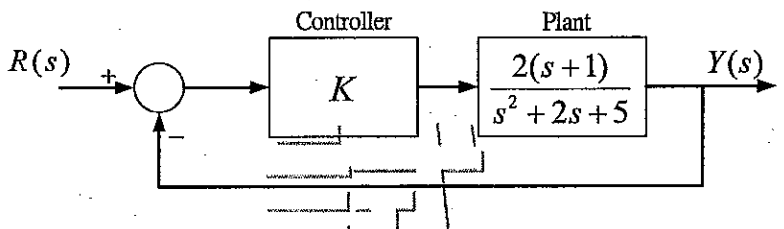
2. (30 pts.) Consider the single-input-single-output system defined by $\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = Cx(t)$, where $u(t)$ is a unit step input, $y(t)$ is the output, and A, B and C are some constant matrices.

(a) Show that $y(t) = Ce^{At}x(0) + CA^{-1}(e^{At} - I)B$ if the matrix A is nonsingular. (10 pts.)

(b) Find the zero-state response of the system if $A = \begin{bmatrix} 0 & 2 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$. (10 pts.)

(c) Determine the equivalence transformation $\bar{x}(t) = Px(t)$ that transforms the original state equations with the matrices in part (b) into the diagonal canonical form, and write down the new state equations, if possible. (10 pts.)

3. (20 pts.) A unity feedback control system with $1 \le K \le 100$ is shown as follows:



(a) Find the value of K to minimize the steady-state error for a unit step input. (10 pts.)

(b) Determine the value of K that will result in zero-overshoot system response with the shortest rising time to a unit step input. (10 pts.)

4. (20 pts.) True or false? Justify you answers. (Give a brief proof or explanation if it is true. Otherwise, give a right correction or counter example if it is false).

(a) If an open-loop system is both controllable and observable, then the closed-loop system obtained through state feedback is also controllable and observable. (5 pts.)

(b) The intersection of the asymptotes of the complete root loci must always be on the real axis of the complex plane. (5 pts.)

(c) The dominant poles of a system are the roots of its characteristic equation near the imaginary axis of the s-plane relative to the other roots of the system because they represent or dominate both transient and steady-state responses. (5 pts.)

(d) A PI controller is essentially a low-pass filter because it has the advantage to attenuate the high-frequency noise and reduce the rising time. (5 pts.)