1. (30 pts.) Consider the dynamic system $\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = Cx(t)$, where

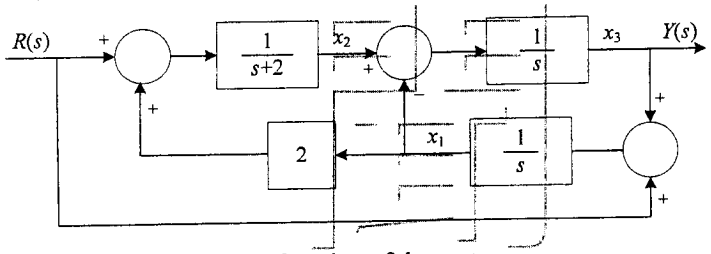
$$A = \begin{bmatrix} -1 & 2 & 2 & -5 \\ 1 & 0 & -2 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & -2 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) (10 pts.) Is the system stable? Why?

(b) (10 pts.) Determine the transfer function from the input $u(t)$ to the output $y(t)$.

(c) (10 pts.) Determine the zero-state response of the system with unit step input.

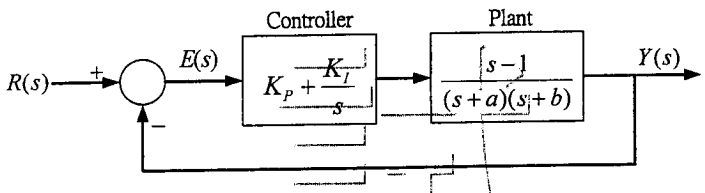
2. (25 pts.) The block diagram of a system is shown in the following figure with the input $R(s)$, the output $Y(s)$ and the three state variables x_1, x_2 and x_3.



(a) (10 pts.) Determine the transfer function of the system.

(b) (15 pts.) Is this system with the given state variables controllable or observable? Why?

3. (25 pts.) A unity feedback control system with a PI compensator is shown as follows:



(a) (10 pts.) If $a = -b$ and $K_I = 0$, find the proportional gain K_P to stabilize the system.

(b) (15 pts.) If $a = b > 0$ and $K_P = K_I$, determine the value of K_P so that the steady-state error $\lim_{t\to\infty} e(t)$ for a unit step input is equal to zero.

4. (20 pts.) True or false? Justify you answers. (Give a brief proof or explanation if it is true. Otherwise, give a right correction or counter example if it is false).

(a) (5 pts.) For a prototype 2nd-order system, the maximum overshoot of its unit-step response and the resonant peak of the magnitude of its frequency response depend on both damping ratio and natural frequency.

(b) (5 pts.) A phase-lag compensator can provide a phase-lag angle and a significant attenuation over the frequency range of interest because it has a frequency response like an integrator over a finite range of frequencies.

(c) (5 pts.) If the characteristic equation of a closed-loop system is $(s+3)(s^2+9) = 0$, then the system is said to be marginally stable. That is to say, the system output will be sustained bounded oscillations for any bounded input.

(d) (5 pts.) The transfer function representation of a linear time-invariant system for input-output relationship is unique, so is the state-space representation.