

科目：工程數學乙 適用：電機所系統組

編號：431

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. Determine the solutions $y(t)$ to the following initial-value problems:

(1) (10%) $y'' + y = \cos \omega t$ ($\omega > 0$), $y(0) = 0$, $y'(0) = 1$

(2) (10%) $y' - 4y = e^{4t}$, $y(0) = 0$

(3) (10%) $y'' - 4y' = 0$, $y(0) = 3$, $y'(0) = 8$

2. (10%) Let $\mathbf{X}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Find a general solution to $\mathbf{X}'(t) = \begin{bmatrix} -8 & -1 \\ 16 & 0 \end{bmatrix} \mathbf{X}(t)$.

3. (10%) Find a general solution $y(x)$ to the following differential equation:

$$x^2 y'' - xy' + y = 0 \quad x > 0.$$

4. (10%) Prove that $\left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 3 \\ -2 \end{bmatrix} \right\}$ is a basis for the null space of the linear

transformation $T: \mathcal{R}^4 \rightarrow \mathcal{R}^2$ defined by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 + x_3 - 3x_4 \\ -2x_1 + 3x_2 - 3x_3 + 2x_4 \end{bmatrix}.$$

5. (15%) Determine if the matrix $A = \begin{bmatrix} -4 & -3 \\ 3 & 6 \end{bmatrix}$ is diagonalizable. If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

6. Let $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \\ 0 \end{bmatrix} \right\}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 7 \\ 4 \\ 7 \end{bmatrix}$.

(1) (10%) Find the orthogonal projection of \mathbf{v} onto W .

(2) (5%) Find the distance from \mathbf{v} to W .

7. Determine if the following statements are true or false.

(1) (5%) Eigenvectors of a matrix that correspond to distinct eigenvalues are orthogonal.

(2) (5%) Every linear operator on \mathcal{R}^n has real eigenvalues.