

科目：工程數學甲 適用：電機所電子組

編號：421

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (30 pts.) Determine $y(t)$ by solving the following differential equations:

(a) (10 pts.) $y^{(4)} + 2y'' + y = 0, \quad y(0) = y'(0) = y''(0) = y'''(0) = 0.$

(b) (10 pts.) $ty \frac{dy}{dt} + y^2 = 32t, \quad y(-1) = 0.$

(c) (10 pts.) $\begin{cases} x'' + 2x = y - x \\ y'' + 2y = x - y, \end{cases} \quad x(0) = 2, \quad x'(0) = y(0) = y'(0) = 0.$

2. (20 pts.) Consider the two signals $y(t) = x(t) * h(t)$ and $z(t) = x(3t) * h(3t)$ (note that the symbol "*" is represented as the convolution integral), and assume that $X(j\omega)$ and $H(j\omega)$ are the Fourier transforms of $x(t)$ and $h(t)$.

(a) (10 pts.) Use Fourier transform properties to show that $z(t)$ can be represented in the form $z(t) = Ay(Bt)$, and determine the values of A and B .

(b) (10 pts.) If $x(t) = \frac{\sin t}{\pi t}$ and $h(t)$ is a unit impulse function, determine and sketch the Fourier transform of $z(t)$.

3. (20 pts.) Consider the following initial-value problem (IVP):

$$y' + ky = te^{-t}, \quad y(0) = 1, \quad k \in \mathfrak{R}.$$

(a) (10 pts.) Find the Laplace transform of the solution for this IVP.

(b) (10 pts.) Determine the solution of this IVP by finding the inverse Laplace transform of part (a).

4. (30 pts.) True or false? Justify your answers. (Give a brief proof or explanation if it is true. Otherwise, give a right correction or counter-example to show that it is false).

(a) (6 pts.) The function $f_1(t)$ is equal to the function $f_2(t)$ if and only if their Laplace transforms are identical.

(b) (6 pts.) A constant multiple of a solution of a linear differential equation is also a solution.

(c) (6 pts.) If the Laplace transform of $f(t)$ is $F(s)$, then the Laplace transform of the function $e^{at} \int_0^t f(\tau) d\tau$ can be written as $\frac{F(s-a)}{s}$.

(d) (6 pts.) The Fourier transforms of real and even signals are also real and even.

(e) (6 pts.) The boundary-value problem $y'' - a^2y = 0, \quad y(0) = y(\frac{\pi}{2}) = 0$, where $a \in \mathfrak{R}$, possesses only the trivial solution.