1. A solid sphere of mass m and radius r rolls without slipping along the track shown in Figure 1. It starts from rest with the lowest point of the sphere at height h above the bottom of the loop of radius R, much larger than r. (a) What is the minimum value of h (in terms of R) such that the sphere completes the loop? (b) What are the force components (F_x and F_y) on the sphere at the point P if $h = 3R$? [Hint: sphere $I = 2mr^2/5$.] (20%)

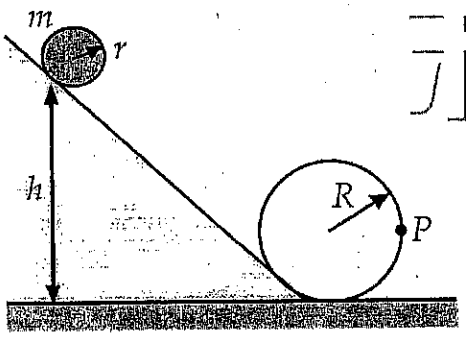

Figure 1

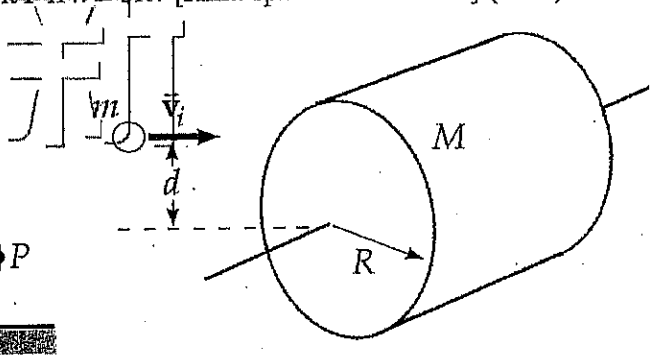

Figure 2

2. A wad of sticky clay with mass m and velocity $\vec{v}_i$ is fired at a solid cylinder of mass M and radius R (Figure 2). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance $d < R$ from the center. (a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. (10%) (b) Is mechanical energy of the clay–cylinder system conserved in this process? Explain your answer. [Hint: cylinder $I = mr^2/2$.] (5%)

3. A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension (Figure 3). Find the frequency of vibration of the system for small values of the amplitude (small θ). Assume that the vertical suspension of length L is rigid, but ignore its mass. (10%)

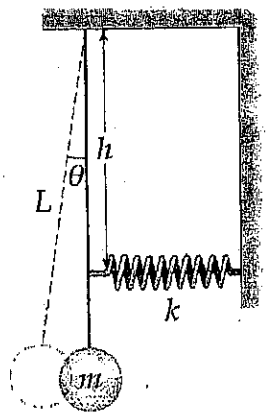

Figure 3

4. A hollow metal cylinder (the cathode) of inner radius r_a and a coaxial cylindrical wire (the anode) of radius r_b (Figure 4). The charge per unit length on the anode is λ, and the charge per unit length on the cathode is $-\lambda$. (a) Find the magnitude of the potential difference between the wire and the cylinder. (b) Find the magnitude of the electric field in the space between cathode and anode is given by where r is the distance from the axis of the anode to the point where the field is to be calculated. (20%)

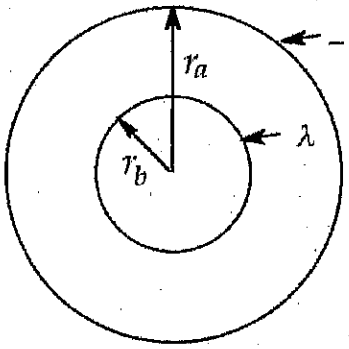

Figure 4

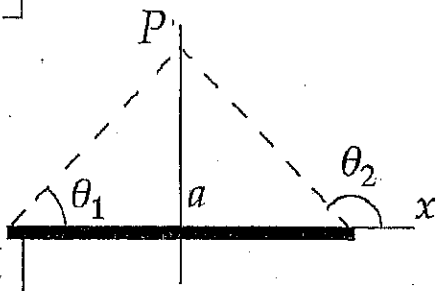

Figure 5

5. Consider a thin, straight wire segment carrying a constant current I and placed along the x axis as shown in Figure 5. (a) Use the Biot–Savart law to find the total magnetic field at the point P, located a distance a from the wire. (b) Assuming that the wire is infinitely long, find the total magnetic field at the point P. (20%)

6. A rod of length L_0 moving with a speed v along the horizontal direction makes an angle θ_0 with respect to the x-axis. (a) Show that the length of the rod as measured by a stationary observer is $L = L_0[1 - (v^2/c^2)\cos^2\theta_0]^{1/2}$. (10%) (b) Show that the angle that the rod makes with the x-axis is given by $\tan\theta = \gamma \tan\theta_0$. These results show that the rod is both contracted and rotated. (Take the lower end of the rod to be at the origin of the primed coordinate system.) (15%) (5%)